

**Document GTPS N°11F
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Severe Tests Statistical Method

**Recommendation for
designing and mastering
reliable pyrotechnic devices**

Contents:

A. INTRODUCTION.....	4
A.1 PURPOSE	4
A.2 BIBLIOGRAPHICAL REFERENCES.....	4
A.2.1 REFERENCE DOCUMENTS.....	4
A.2.2 OTHER DOCUMENTS	5
A.3 SPECIFIC TERMINOLOGY	5
B. PART ONE: RECOMMENDATION FOR DESIGNING AND MASTERING RELIABLE PYROTECHNIC DEVICES.....	6
B.1 SCOPE	6
B.2 METHODOLOGY BY PHASE	6
B.2.1 GENERAL RULES.....	6
B.2.2 FEASIBILITY.....	7
B.2.2.1 Purpose	7
B.2.2.2 Tasks.....	7
B.2.3 PRE-PROJECT.....	8
B.2.3.1 Purpose	8
B.2.3.2 Tasks.....	8
B.2.4 DEVELOPMENT.....	9
B.2.4.1 Purpose	9
B.2.4.2 Tasks.....	9
B.3 PRESENTATION OF THE STATISCAL METHODS USED	11
B.3.1 PURPOSE OF METHODS	11
B.3.2 CONDITIONS FOR IMPLEMENTING METHODS	11
B.3.2.1 Definition of test sampleS.....	11
B.3.2.2 TEST REPRODUCIBILITY.....	12
B.3.2.3 Prerequisites.....	13
B.4 COMPARISON OF METHODS FOR ASSESSING PYROTECHNIC DEVICES RELIABILITY	14
C. SECOND PART: IMPLEMENTATION OF THE "SEVERE TEST" METHOD.....	15
C.1 NOTATION CONVENTIONS	15
C.2 AIM OF THE METHOD	15
C.2.1 PRINCIPLE OF THE METHOD	16
C.2.2 PREREQUISITE CONDITIONS.....	16
C.2.3 SEVERITY PARAMETER DETERMINATION.....	17
C.2.4 COEFFICIENT OF VARIATION DETERMINATION.....	18
C.2.4.1 GENERAL OBSERVATIONS.....	18
C.2.4.2 ELEMENTARY COEFFICIENTS OF VARIATION (cvi) GATHERING	18
C.2.4.3 CORRECTED COEFFICIENT OF VARIATION (cvc) DETERMINATION	19
C.2.4.4 RECOMMENDATIONS	19
C.2.5 SEVERE TESTS PLAN DESIGN	21
C.2.5.1 DEFINITION OF LEVELS	21
C.2.5.2 SEVERITY COEFFICIENT DEFINITION	21
C.2.5.3 MULTIPLIER COEFFICIENT CALCULATION	22
C.2.5.4 DIVISOR COEFFICIENT CALCULATION.....	24
C.2.5.5 RECOMMENDATIONS	25
C.2.5.6 SPREADSHEET.....	26
C.2.6 IMPLEMENTATION OF THE SEVERE TESTS PROGRAMME	26
C.2.6.1 FAILURE-FREE PROGRAMME.....	26
C.2.6.2 PROGRAMME WITH FAILURES	27
C.2.6.3 RECOMMENDATION	29

GTPS N°11F

D. APPLICATION EXAMPLE	30
D.1 OBJECTIVE.....	30
D.2 IDENTIFYING THE SEVERITY PARAMETER	30
D.3 DETERMINING THE COEFFICIENT OF VARIATION CVG.....	31
D.4 DESIGNING THE SEVERE TESTS PROGRAMME	32
D.5 IMPLEMENTATION AND EXPLOITATION OF SEVERE TESTS	32
D.5.1 CASE 1: NO FAILURE OBSERVED	33
D.5.2 CASE 2: ONE FAILURE OBSERVED (DEGRADED CASE)	33
D.5.3 CASE 3: TWO FAILURES OBSERVED (FAILURE OF THE SEVERE TESTS PROGRAMME).....	34
E. CONCLUSION	35
F. ACKNOWLEDGEMENTS	35
ANNEXES:	36
ANNEXE 1: CHECKING HOW REPRESENTATIVE THE TESTED SAMPLES ARE.....	37
ANNEXE 2: CV WEIGHTING USING EXPERIMENTAL DESIGN	38
ANNEXE 3: TREATMENT OF TEST PROGRAMMES WITH FAILURES.....	40
ANNEXE 4: ANALYSIS AND CONSTRUCTION OF CVG	41
ANNEXE 5: MINIMUM LEVEL OF SEVERITY COEFFICIENT.....	44
ANNEXE 6: CONSTRUCTION OF SEVERE TESTS PROGRAMME.....	45
ANNEXE 7: IMPLEMENTATION AND USE OF THE SEVERE TESTS PROGRAMME	46
ANNEXE 8: SENSITIVITY ANALYSIS OF THE SEVERITY COEFFICIENT	47

GTPS N°11F

A. INTRODUCTION

A.1 PURPOSE

This document is a translation of the French document GTPS 11F.

The first part of this recommendation is intended for designers to enable them to create and master the manufacturing of reliable pyrotechnic products. It should provide a basis for discussions between customers and suppliers whenever a contract binding them requires reliability specifications. It covers:

- the type of design phases,
- the procedures for ensuring reliability for each of these phases,
- a presentation of various statistical methods available to designers including:
 - suitable methods for each of the design phases,
 - the advantages and drawbacks of each of the methods explained.

The second part of this recommendation presents the procedure for implementing the "severe tests" statistical method.

A.2 BIBLIOGRAPHICAL REFERENCES

A.2.1 REFERENCE DOCUMENTS

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21. Technical document **A5-NT-1-X-542-ASAI** du 1990-08-28 : « Systèmes pyrotechniques Marges de dimensionnement et de performances ».

A.3 SPECIFIC TERMINOLOGY

In order to avoid any misunderstanding in this recommendation, it was decided that the following concepts should be clearly defined:

- **Design:** Based on expressed needs and existing knowledge, it is a creative activity that leads to product definition compliant with these needs and that is processable,
- **Product:** a term covering any item resulting from a production operation or any service provided such as production of components (raw materials, semi-finished or finished products, ingredients, parts, components, hardware, systems, etc.),
- A **functional parameter** is a quantifiable physical magnitude, associated with the product, whose value affects the success-failure criteria during their implementation,
- The **success** or **failure criterion** is a way of characterising the response of the product to stress,
- The **operating threshold** of a product for a given reliability **R** is defined as being the value of the functional parameter for which the probability of success is equal to **R**.

B. PART ONE: RECOMMENDATION FOR DESIGNING AND MASTERING RELIABLE PYROTECHNIC DEVICES

B.1 scope

This document is intended for all industrial designers who need to respond to a formal quantitative reliability requirement of a pyrotechnic device. It covers:

- In conformity with **[10]**, design activities, including feasibility, pre-project and development phases, during which reliability must be taken into account to define a product which can be manufactured at optimised cost.
- Continuous design activities to improve the reliability of a given product.

It applies to products using pyrotechnic devices defined in the document cited at paragraph 1 at section A.2.1 (one-shot devices).

B.2 METHODOLOGY BY PHASE

B.2.1 GENERAL RULES

1. Determine the objectives to be achieved in terms of performance, characteristics, costs and timeframe,
2. Integrate and manage reliability during the project design phases,
3. Have a systematic dialogue structure between the parties concerned,
4. Ensure consistency of the objectives with:
 - Actions planned,
 - Results obtained,
5. Ensure that the technical and human resources used correspond to the product being designed.

Associated with these rules are certain tasks such as management, calculation, analysis or testing. In particular, they are due to the necessary iteration between the dimensioning of the product and its reliability expressed in terms of margins and design factors.

B.2.2 FEASIBILITY

B.2.2.1 PURPOSE

The purpose of this phase is to show if the stated requirements can be met, by detailing the possible concepts, technological routes and architectures. Such requirements are generally expressed in terms of the mission objectives, information concerning the operational environment (life cycle with associated environmental conditions) and reliability objectives.

It has to work towards establishing the reliability requirements to be included in the functional performance specifications, and possible reliability management requirements.

B.2.2.2 TASKS

For each proposed technological solution, the tasks to be accomplished are:

- Preliminary risk/hazard analysis,
- Risk assessment by:
 - literature survey and/or experience acquired with similar products, especially regarding anomalies or incidents encountered; reliability database research,
 - computed simulation to gain quantitative and qualitative understanding of the phenomena involved and to highlight certain critical design features,
 - use of an experimental design to establish the predominant parameters, their sensitivity on performance and their interactions,
 - implementation of one of the methods recommended in the table section B.4 to estimate the mean for certain specific parameters,
- Appraisal of critical points highlighted for each solution, and comparison of solutions with respect to the stated requirements.

By the end of this phase, qualitative assessment criteria should comprise the input data necessary to start the next phase. Accordingly, they should be set down in the functional performance specifications in the chapter on reliability requirements.

B.2.3 PRE-PROJECT

B.2.3.1 PURPOSE

The purpose of this phase is to investigate the possible approaches at the end of the feasibility study so as to suggest what can be developed.

It enables the preliminary product definition file to be prepared in accordance with the reliability requirements of the functional performance specifications established during the previous phase.

B.2.3.2 TASKS

For each solution considered feasible:

- **Modelling:** draw up a reliability block diagram in order to establish product architecture and identify the interfaces concerned by the reliability study. This approach is used to define the "product" tree diagram whose level of breakdown stops at the basic components with measurable characteristics,
- **Allocation:** distribute the overall reliability objective among items on the tree, allocating a predicted reliability objective to each of the itemised components and interfaces to indicate the probability of the function being fulfilled for each component, allowing for life cycle and/or its life time,
- **Analysis:** for each component listed, perform a Failure Mode, Effects, and Criticality Analysis (FMECA) to highlight the points considered critical based on:
 - existing databases and/or feedback on similar components,
 - possibly, and depending on the products developed, a specific experiment using the method(s) specified in the table section B.4 to first confirm the initial assessment of the average m (see section B.2.2.2), and secondly to provide an initial estimate of the standard deviation σ of the dispersion around the mean value.
- **Forecast:** using the reliability block diagram model, piece together partial assessments in accordance with the product tree to assess how the proposed solution matches the requirement.
- **Validation plan:** draw up a pre-project development - reliability product plan to estimate what technical work is necessary to develop the product satisfactorily in terms of cost and timeframe.
- **Trade-off:** considering all the solutions, choose the one which best meets the stated requirement, and which will be developed in the subsequent phase, while justifying why the other solutions are rejected.

B.2.4 DEVELOPMENT

B.2.4.1 PURPOSE

The purpose of this phase is:

- to draw up the product definition file to meet the reliability requirements as expressed in the Technical Specifications,
- to validate the design using the results of theoretical studies, tests, and exploitation of technical fact,
- to prepare production and operational phases, specifying which procedures will be necessary for ensuring reliability during these two phases.

B.2.4.2 TASKS

For the adopted solution:

- Conduct a reliability predicted study in order to:
 - rework and refine the previous reliability block diagram,
 - optimise the environmental constraints applied to each component,
 - possibly, update the reliability allocations and negotiate reliability requirement,
- Identify feared events by a fault tree analysis. Deductive analysis is a statistical analysis which does not take the sequential aspect of events into account. Limits inherent in implementing fault tree analysis are:
 - To correctly define the feared event (origin of the tree)
 - To define elementary events,
 - To ensure the independence of the elementary events listed,
- Carry out FMECA for each elementary event listed,
- Define all the solutions needed to meet the required levels of reliability, by means of:
 - studies and tests up to the product qualifying phase (design reliability), by implementing the methods recommended in the table section B.4,
 - manufacturing and acceptance procedures (manufacturing reliability),
- *A posteriori*, check and assess the independence of the events,

GTPS N°11F

- Undertake long-term actions to ensure reliability throughout the life time of the product. In particular, define the ageing programme to be conducted in order to :
 - ensure that the assumed level of reliability has been attained,
 - assess what advance warning is required to prevent or overcome a possible long-term failure,
 - upgrade the databases, especially those used for reliability analysis during development.

The associated sampling policy should be consistent with the operational needs.

At the end of the development phase, the product design shall meet reliability objectives.

The development phase is finalised by the approval dossier, qualification and/or certification report (definition file and supporting evidence, industrial file).

B.3 PRESENTATION OF THE STATISCAL METHODS USED

B.3.1 PURPOSE OF METHODS

The purpose of these methods is to:

- Characterize the distribution of product operation thresholds by sensitivity tests (either sequential or simultaneous).
- Check the appropriate probability distribution for these operating thresholds,
- Use this distribution to assess a probability of success or failure during operation of the product tested for a given confidence level.

B.3.2 CONDITIONS FOR IMPLEMENTING METHODS

B.3.2.1 DEFINITION OF TEST SAMPLES

The definition of test samples has to take the three following points into account:

1. Nominal definition of test specimen:

- The nominal definition of test specimen complies with a Definition File and the sample is representative of a given population (see Appendix 1).
- The test specimen can be:
 - A functional object (e.g., an initiator, the couple formed by a shear and the rod to be cut, etc.),
 - A defined quantity of a product.

2. Definition of the population:

- The test specimen belongs to a clearly identified population.
- It is recommended that a homogeneous batch, manufactured at the same time and place, using the same raw materials, methods and personnel, be used, and in any event, in accordance with the methods and equipment defined (see Appendix 1).

3. Definition of the test sample:

It is chosen from the population following a sampling plan defined by:

- The type of test,
- The particular sampling scheme required to ensure the validity of the test results,
- The size of sample to be tested. It depends on the method used, as explained in the table section B.4. However it is recommended that a reserve for additional specimens is established for contingencies.
- The relationship between the tests results and the test acceptance criteria.

B.3.2.2 TEST REPRODUCIBILITY

Test reproducibility has to take the following four points into account:

1. Identification of test support equipment:

- Consumable test support equipment compliant with a Definition File,
- Reusable test support equipment for which compliance with a Definition File and stability of functional characteristics will be checked.

2. Identification of test facilities:

- Environmental conditions [9],
- Power sources,
- Calibrated measuring equipment.

3. Control of stresses applied to the specimens:

- The uncertainty of stresses applied has to be less than the assumed standard deviation for the population.

4. Control of test conditions:

- Stable environmental and test conditions during a test sequence,
- Representative conditions and / or test specimens from the actual configuration (confinement, critical diameter, heat exchange ...),
- Test facilities,
- Procedures,
- Personnel.

B.3.2.3 PREREQUISITES

1. Choice of the functional parameter:

It must meet the following criteria:

- To be adjustable,
- To behave in a known and continuous manner in the field to be investigated.

2. Choice of the success/failure criterion:

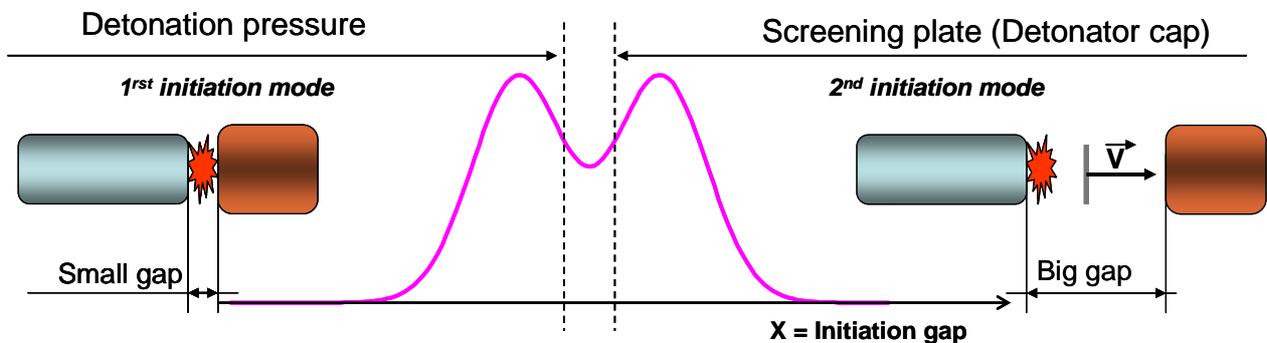
- It must be clearly defined, after analysis of all possible responses of the product studied.
- It is necessary to understand how the probability of success or failure varies according to the variation rate of the chosen functional parameter.

3. Assumptions:

It is assumed that:

- The resolution of the functional parameter for the test should be approximately 1/10 of the initial evaluation of the estimated standard deviation.
- The functional threshold of the selected functional parameter is a random variable.
- The density probability of this random variable follows a normal (1) or a log normal (2) law. It should take into account experience feedback.

(1) NOTE: Concerning the assumption of normality, it shall be ensured that the selected functional parameter is led by **only one physical phenomenon** in the field trials. Indeed, some cases may be governed by several physical phenomena that lead to multimodal statistical laws, like the initiation gap between an explosive relay and a detonator:



(2) NOTE: If a log-normal probability density function is used, a change of variable will be done in order to reduce the studied case to a normal statistical law.

B.4 COMPARISON OF METHODS FOR ASSESSING PYROTECHNIC DEVICES RELIABILITY

Table 1 below lists the advantages and drawbacks of each of the statistical methods.

Method	No. of tests	Advantages	Drawbacks
Probit GTPS 11A (see para. 2 of section A.2.1)	≥ 72	Non sequential test Possible to adjust levels during tests Best estimator of standard deviation	Define at least 5 levels Significant risk that the method will fail (estimated at 16%), even under ideal test conditions.
One-shot GTPS 11B (see para. 3 of section A.2.1)	≥ 30	All test results can be used Choice of initial test value does not alter results accuracy Convergence toward the mean is assured and very fast for a small sample tested: <ul style="list-style-type: none"> • possibly poorly known, • whose probability distribution law is unimodal 	Sequential test involving management of test constraints with levels unknown in advance
Bruceton GTPS 11C (see para. 4 of section A.2.1)	≥ 30	Provides statistical estimators for mean and standard deviation, with good precision for the mean	Sequential test involving management of test constraints, but with a fixed pitch Results depend on the pitch value
Severe tests GTPS 11F (See Part Two of this document)	≥ 1 ≤ 10	Method suitable to confirm functional margins with respect to its nominal operating point, with less than 10 trials. Analytical approach taking into account the contents of the FMECA as a complementary tool. Enables one failure in implementing the severe tests plan, through: <ul style="list-style-type: none"> • either a degraded reliability assessment (compared to the initial value intended) • or an increase in the number of specimens tested 	Requires knowledge of the coefficients of variation of predominant parameters Results depend closely on the coefficients of variation associated with widely scattered parameters. Does not provide the distribution of the functional parameter being tested.

Table 1

C. SECOND PART: IMPLEMENTATION OF THE "SEVERE TEST" METHOD

C.1 NOTATION CONVENTIONS

- . **CVc** = corrected coefficient of variation of the predominant functional parameter
- . **CVg** = global coefficient of variation of the predominant functional parameter
- . **CVi** = elementary coefficient of variation of the functional parameter X_i
- . **CV0** = elementary coefficient of variation of the predominant functional parameter
- . **I** = incomplete BETA function
- . **I⁻¹** = Inverse incomplete BETA function
- . **N** = standard Normal distribution
- . **N⁻¹** = inverse function of the standard Normal distribution
- . **Kd** = severity coefficient, divisor
- . **Km** = severity coefficient, multiplier
- . **m** = mean of a population distribution
- . **n** = number of tests
- . **R** = reliability to be assessed (at reference level X_{ref})
- . **R_s** = reliability at severity level X_{sev}
- . **s** = estimator of the standard deviation of a population distribution
- . **U_s** = value of the standardized variable corresponding to the severe level
- . **U_R** = value of the standardized variable corresponding to the reliability level R to be assessed
- . \bar{x} = estimator of the mean of the probability distribution of a population
- . **X_{sev}** = severity level at which the severe tests will be carried out to validate that the reliability objective has been achieved
- . **X_{nom}** = nominal level of the predominant functional parameter
- . **X_{ref}** = level of the nominal requirement to which the reliability objective corresponds
- . **σ** = standard deviation of the probability distribution of a population
- . **1 - α** = confidence level required for assessment of reliability

C.2 AIM OF THE METHOD

This method is used to validate compliance with a reliability objective for a limited number of tests of a one-shot device.

It is applicable to pyrotechnic products in particular.

The method can be used for each of the product design phases (feasibility, pre-project, development or qualification), as defined in part B of this document.

C.2.1 PRINCIPLE OF THE METHOD

In order to assess a specific reliability objective, the principle of the method is to determine a severity coefficient to be applied to the predominant functional parameter of a pyrotechnic device and to demonstrate that this device will operate "without failure" via tests at the level of severity.

Under specific conditions (dealt with later in C.2.6.2), it is possible to incorporate the concept of failure when analysing the results.

C.2.2 PREREQUISITE CONDITIONS

Functional, physical and process analyses lead to an understanding and control of the main parameters.

For a given confidence level, the severe tests plan will be applied to assess the reliability of one elementary function (*) of the tested product (cutting, ignition, structural strength, etc.).

(*) **NB:** as for all statistical methods, the severe tests method shall only be used to assess the reliability of one elementary function of the product and not its overall reliability.

Application of the severe tests method to an elementary function requires prior knowledge or determination of the following parameters:

- A predominant functional parameter X (for which the distribution of functional thresholds shall be Normal): see how to process in C.2.3,
- The elementary coefficients of variation CV_i of the functional parameters that act on the studied elementary function (see how to determine them in C.2.4),
- Reliability level R to be assessed for the elementary function,
- Confidence level $1-\alpha$ for which reliability is assessed (*),
- Knowledge of existing differences between the prototype production process and the serial manufacturing process. It is recommended for the prototype process to be as similar as possible to the manufacturing one (See B.3.2.1 and **Annexe 1**).

(*) **NB:** in practice, $1-\alpha$ is commonly between 90% and 60% according to customer.

Annexe 9 provides a method to re-assess reliability level at a new given confidence level from an already carried out severe tests plan (confidence level and reliability are known).

C.2.3 SEVERITY PARAMETER DETERMINATION

From a physical analysis and/or a functional description of the system, the functional parameters of the product shall be determined.

Those which affect the elementary studied function of the product shall be selected:

- These parameters shall be mutually independent (this independency has to be verified).

Among the main parameters identified, a functional parameter which is considered as predominant shall be chosen. This parameter (to be hardened) shall:

- have a significant effect on the performance to be achieved,
- be quantifiable,
- be measurable,
- be easily adjustable.

Examples of parameters representative of an elementary function:

- "Tensile strength" of a tie-rod or "initial speed" provided by an separating device,
- "Heating energy" or "pressure" necessary to ignite a pyrotechnic device,
- "Cross-section" of a tie-rod to be cut or "powder mass" of pyrotechnic shears,
- Ignition "temperature" of pyrotechnic materials; in this case, particular attention should be paid to the unit used: The Kelvin is the only unit allowed for temperature measurement (only referable and measurable units for temperature),
- Etc...

The other functional parameters which affect the elementary function will be taken into account when determining the corrected coefficient of variation to be applied to the predominant parameter:

- If there are enough of them (≥ 5), their overall contribution to the distribution of the operational threshold of the predominant parameter can be considered to be a Normal distribution within their working and test ranges, regardless of their individual distributions,
- If there are too few of them (< 5), their overall contribution to the predominant parameter shall be normally distributed within their working and test ranges.

GTPS N°11F

NB 1: As far as possible, these assumptions should be verified by:

- relying on data bases,
- state of the art,
- tests,
- similarity (identical or derivative product),
- numerical simulation,
- Etc...

NB 2: Some important functional parameters can be frozen at their upper or lower bounds of use within the severe tests plan, and hence not included when calculating the corrected coefficient of variation of the predominant parameter.

C.2.4 COEFFICIENT OF VARIATION DETERMINATION

C.2.4.1 GENERAL OBSERVATIONS

In the rest of this text, the statistical estimators of a sample drawn from the mother population of mean m and standard deviation σ , will be noted respectively by \bar{x} and s .

The coefficient of variation is therefore defined as the ratio of the estimators of the standard deviation to the mean performance of the functional parameter in question:

$$CV = \frac{s}{\bar{x}}$$

Calculation of CV should be representative of the performance of the system or of the component under specified conditions (operating conditions and environment, lifetime ...).

Without knowledge of the mean \bar{x} and the standard deviation s of the functional preponderant parameter, it is necessary to first determine the coefficients of elementary variations (**CVi**) of the other influential parameters on the studied performance.

C.2.4.2 ELEMENTARY COEFFICIENTS OF VARIATION (CVI) GATHERING

The elementary coefficients of variation of the influential parameters for a pyrotechnic device can be obtained from:

1. Statistical analysis of tests (One-shot, Probit, Bruceton, or other methods),
2. Experience acquired with similar equipment, assigning an a priori distribution to the parameter (obtaining the coefficient of variation by similarity),
3. Numerical simulations,
4. Experience with the manufacturing and inspection processes and facilities (tolerance interval assimilated to a ± 3 standard deviations range, which means controlling all the significant parameters of the process),
5. Statistical analysis of design of experiment

C.2.4.3 CORRECTED COEFFICIENT OF VARIATION (CVC) DETERMINATION

Determination of the predominant parameter coefficient of variation noted **CV0** and obtained as above, shall be corrected to take into account:

- expected scatterings not included in other important parameters,
- parameters which have not been included in the previous estimates:
 - ageing,
 - change of batch (powder, raw materials, components, etc.),
 - mechanical and climatic environments,
 - etc.

Without knowledge of the contributions (weights) of various parameters on the variability of the performance study, we calculate the corrected coefficient of variation **CVc** using the following simplified formula:

$$CVc = \sqrt{CV0^2 + CV1^2 + \dots + CVi^2 + \dots}$$

NB: If a mathematical model of the studied performance has been established by design of experiment or CAE (Computer Aided Element) (see example in **Annexe 2**), then the coefficients of variation **CVi** can also be combined with their weights.

C.2.4.4 RECOMMENDATIONS

RECOMMENDATION 1: Fixed margin for the coefficient of variation

It is recommended to apply a fixed margin factor of 10% to the coefficient of variation obtained, in order to compensate for:

- incomplete knowledge of the effect of some functional parameters,
- incomplete knowledge of some physical phenomena (actual distribution laws, important parameters which cannot be measured, unknown environment, low number of tests for determining the coefficient of variation, etc.).

Hence we obtain the global coefficient of variation:

$$CVg = 1.1 \times CVc$$

RECOMMENDATION 2: Severe tests method application field

The method is not relevant in the following cases:

- **CV_g** is very low (< 3 %):
 - either the CV does not take all the important factors into account:
 - it is recommended in this case to rely on the functional knowledge of the system (Functional analysis, FMECA) to find the missing information,
 - or the CV is actually very small (as it is the case for a simple and well controlled system), and the method leads to a "very small" severity coefficient (and hence not significant):
 - it is recommended to apply a minimum severity coefficient of 1.2 (this minimum level guarantees a sufficient margin between the nominal mean level and the severe level: see supporting evidence in **Annexe 5**).
- **CV_g** is very high (> 15 %), which is inconsistent with the assessment of a high level of reliability using this method:
 - it is not possible to define a severity coefficient consistent with the required level of reliability whatever the number of test samples (Annexe 4),
 - the very high severity coefficient might not be consistent with the product under investigation.

Over 15%, it is generally assumed that the definition and the manufacturing process are insufficiently well controlled to meet the requirements. This method can only be used under waiver after deeper investigations.

RECOMMENDATION 3: Inclusive coefficient of variation

In the case where no evidence for the coefficient of variation is available, it is recommended to use an inclusive global coefficient of variation, established on unanimously acknowledged feedback.

If there is no feedback available, an inclusive global coefficient of variation is taken: **CV_g = 15%**.

C.2.5 SEVERE TESTS PLAN DESIGN

The design logic of the severe tests plan is described in the flowchart in **Annexe 6**.

The aim is to determine a severity level for the predominant functional parameter for which the tests have to be carried out.

C.2.5.1 DEFINITION OF LEVELS

Nominal level (X_{nom}): this is the nominal value of the predominant parameter

Reference level (X_{ref}): this is the value of the predominant parameter corresponding to the reliability assessment (possible margins already taken on the nominal level)

According to the case, the reference level could be:

- the nominal level of the predominant functional parameter,
- the nominal value of the predominant functional parameter with a margin coefficient specified by the customer or by standards,
- an upper or lower bound of a deterministic requirement,
- the level corresponding to the limits of the tolerance interval of the nominal level,
- the level corresponding to the limits at ± 3 standard deviations from the nominal level,
- ...

Severe level (X_{sev}): the severe tests are carried out at this value of the predominant parameter.

C.2.5.2 SEVERITY COEFFICIENT DEFINITION

The severity coefficient is defined as the ratio of the severe level to the reference level.

This coefficient can take one of the two following forms:

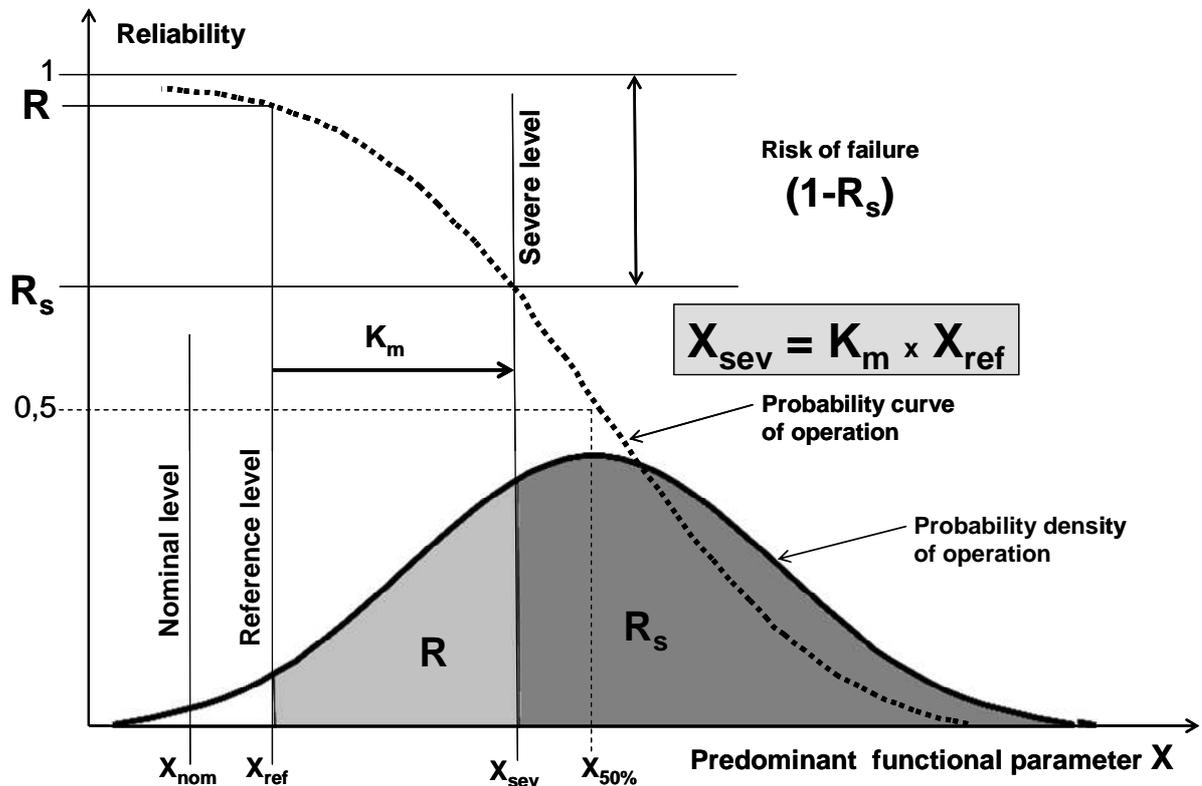
- **Case 1:** Severity coefficient multiplier **Km**: the severity level shall be compared with a lower reference level,
- **Case 2:** Severity coefficient divisor **Kd**: the severity level shall be compared with a higher reference level,

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The methods for calculating the severity coefficients for the "multiplier" and "divisor" cases are shown in C.2.5.3 and C.2.5.4:

- The formulae developed assume that the severe tests plan are carried out without failure, this being the nominal case for application of the severe tests method.
- If a failure appears, more general formulae are shown in **Annexe 3** which take such a failure into account. In this case, the procedure to be followed is described in C.2.6.2 and in the flowchart in **Annexe 7**.

C.2.5.3 MULTIPLIER COEFFICIENT CALCULATION



The use of the Standard Normal Law [N(0,1)] requires to transform the predominant functional random variable into a reduced Normal variable.

If $N(X)$ is the Normal distribution function of X then:

- at the reference level, the z-score is written as:

$$U_R = \frac{X_{ref} - X_{50\%}}{s}$$

- at the severe level, the z-score is written as:

$$U_s = \frac{X_{sev} - X_{50\%}}{s}$$

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After solving the above equation system, and given that $X_{sev} = K_m \times X_{réf}$, we get:

$$K_m = \frac{X_{50\%} + U_s \times s}{X_{50\%} + U_R \times s}$$

Given that the global coefficient of variation is $CVg = \frac{s}{X_{50\%}}$, K_m becomes:

$$K_m = \frac{1 + U_s \times CVg}{1 + U_R \times CVg}$$

U_R is the value of the reduced centralised variable corresponding to the probability of failure $1 - R$ for the reference value $X_{réf}$:

$$R = 1 - N(U_R) = N(-U_R)$$

U_s is the value of the reduced centralised variable corresponding to the observed probability after n successful tests at a severe level, for a fixed confidence level of $1 - \alpha$. This value of U_s is determined from the binomial distribution, for which the formula, given zero failures at the severe level, is:

$$\text{Prob (parameter } X > X_{sev}) = \alpha^{1/n} = R_s = 1 - N(U_s) = N(-U_s)$$

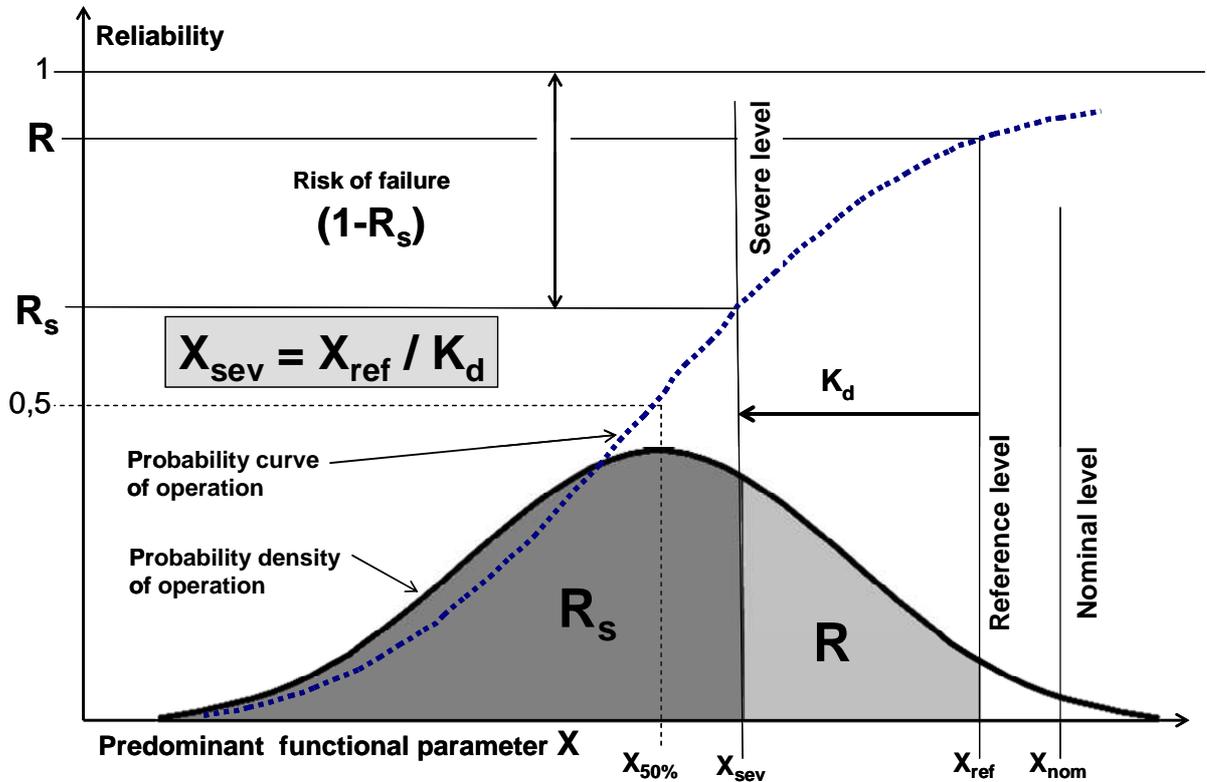
The expression for K_m can finally be written as:

$$K_m = \frac{1 + N^{-1}(1 - \alpha^{1/n}) \times CVg}{1 - N^{-1}(R) \times CVg} = \frac{1 - N^{-1}(\alpha^{1/n}) \times CVg}{1 - N^{-1}(R) \times CVg}$$

With $N(x)$: Normal law of x :
$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du$$

C.2.5.4 DIVISOR COEFFICIENT CALCULATION

The procedure is the same as the multiplier coefficient determination one, i.e.:



The use of the Standard Normal Law [N(0,1)] requires to transform the predominant functional random variable into a reduced Normal variable.

If $N(X)$ is the Normal distribution function of X then:

- at the reference level, the z-score is written as:

$$U_R = \frac{X_{ref} - X_{50\%}}{s}$$

- at the severe level, the z-score is written as:

$$U_s = \frac{X_{sev} - X_{50\%}}{s}$$

After solving the above equation system, and given that $X_{ref} = K_d \times X_{sev}$, we get:

$$K_d = \frac{X_{50\%} + U_R \times s}{X_{50\%} + U_s \times s}$$

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Given that the global coefficient of variation is $CVg = \frac{s}{X_{50\%}}$, K_d becomes:

$$K_d = \frac{1 + U_R \times CVg}{1 + U_s \times CVg}$$

U_R is the value of the reduced centralised variable corresponding to the probability of failure $1 - R$ for the reference value X_{ref} :

$$R = N(U_R)$$

U_d is the value of the reduced centralised variable corresponding to the observed probability after n successful tests at a severe level, for a fixed confidence level of $1 - \alpha$. This value of U_s is determined from the binomial distribution, for which the formula, given zero failures at the severe level is:

$$\text{Prob (Parameter } X < X_{sev}) = \alpha^{1/n} = R_d = N(U_d)$$

The expression for K_d can finally be written as:

$$K_d = \frac{1 + N^{-1}(R) \times CVg}{1 - N^{-1}(1 - \alpha^{1/n}) \times CVg} = \frac{1 + N^{-1}(R) \times CVg}{1 + N^{-1}(\alpha^{1/n}) \times CVg}$$

With $N(x)$: Normal law of x : $N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du$

C.2.5.5 RECOMMENDATIONS

RECOMMENDATION 4: Maximum severity coefficient

It is recommended to check that the calculated severity level has an acceptable level of risk (cf section B.3.2.3). Otherwise:

- reduce the severity coefficient:
 - by increasing the number of severe firings,
 - or by downgrading the reliability to be evaluated,
- or re-design the object (for example by increasing margins).

RECOMMENDATION 5: Sensitivity analysis

Before proceeding with the severe tests programme, it is recommended that a severe level sensitivity analysis be carried out, varying the level of reliability to be assessed.

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The method does not apply if the variation of the severe level X_{sev} is not significantly higher than the accuracy level adjustment at a significantly different level of reliability. In fact the same severe level would evaluate very different reliability levels (See example in **annexe 8**).

RECOMMENDATION 6: Physical domain of tests

Before proceeding with the severe tests programme, it is recommended to check that the physical phenomenon of operation at severe level is the same as the one at nominal level (for example: a change of phase due to heat input, cf section B.3.2.3).

C.2.5.6 SPREADSHEET

A spreadsheet running French version of MICROSOFT EXCEL[®] is available on the web site GTPS (<http://www.afpyro.org/gtps>): this can be used with or without a failure.

C.2.6 IMPLEMENTATION OF THE SEVERE TESTS PROGRAMME

This section deals with the test programme implementation and its results exploitation.

Two cases may arise:

- The test programme evolves as expected: no failure is observed,
- The test programme does not evolve as expected: one or more failures are observed.

C.2.6.1 FAILURE-FREE PROGRAMME

The result is as expected: reliability is thus properly assessed at the level required by the test programme.

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C.2.6.2 PROGRAMME WITH FAILURES

The causes and procedures to find solutions are given in the table below:

Possible causes and significance		Solving process	Comments
Implementation of test programme	Elements external to the product having led to failure of the samples concerned: - test implementation, - test conditions, - etc.	Execute the following sequence: - check that these elements are not conducive to success, - exclude failed and doubtful trials, - resupply samples, - terminate the test programme.	It is essential to demonstrate that the specific causes identified: - are responsible for the observed failures, - are not responsible for any increase in variability which might cause a bias in favour of the observed successes. If this is not the case, remove all the incriminated samples (successes <u>and</u> failures).
Design of test programme	The severity coefficient is too high for the product being tested: - number of trials too small, - reliability objective too high, - CVg too high.	Reduce severity coefficient: check CVg and increase number of trials, And/or Renegotiate reliability objective.	Previous successes remain counted as successes in the new programme for a reduced severity coefficient.

GTPS N°11F

Possible causes and significance		Solving process	Comments
Manufacture of test specimens	Specific manufacturing cause(s) which led to failure of the concerned samples: - tolerances not met, - defective material, - defective heat treatment, - etc.	Execute the following sequence: - check that these elements are not conducive to success - exclude failed and doubtful trials, - resupply samples, - terminate the test programme.	It is essential to demonstrate that the specific causes identified: - are responsible for the observed failures, - are not responsible for any increase in variability which might cause a bias in favour of the observed successes. If this is not the case, remove all the incriminated samples (successes <u>and</u> failures).
Product design	The product does not meet the expected level of reliability: error in design margins (wrong functional modelling, etc.).	Execute the following sequence: - renegotiate a reliability objective consistent with system requirements, - reuse the severe tests programme at a new level of reduced reliability (after taking any observed failures into account), using the calculation method given in annexe 3 , Or Redesign the product.	

GTPS N°11F

Possible causes and significance		Solving process	Comments
Residual statistical risk	Single failure case: statistically the probability of just one failure is low but is still possible.	Complete with additional trials to achieve the reliability objective (taking failure into account), using the calculation method given in annexe 3 , Or Evaluate a lower reliability (with respect to the target objective), using the calculation method given in annexe 3 (to be renegotiated at system level).	This risk is envisaged as the last option : all other causes previously implicated must be eliminated. ----- If the residual statistical risk assumption is incorrect, there is the risk of a second failure in the additional severe trials.
	Several failures: statistically the probability of several failures is low and this reveals some problem further upstream (causes as above).	Statistical justification is not credible. ----- Redo: - the entire severe tests programme And/or - the design.	All previous causes must be re-examined.

C.2.6.3 RECOMMENDATION

RECOMMENDATION 7: Failure analysis

As soon as a failure occurs it is recommended to stop the trials and to analyse the cause of this failure. The subsequent trials should then be resumed after correction of the problem; this will avoid further failures due to the identified cause.

D. APPLICATION EXAMPLE

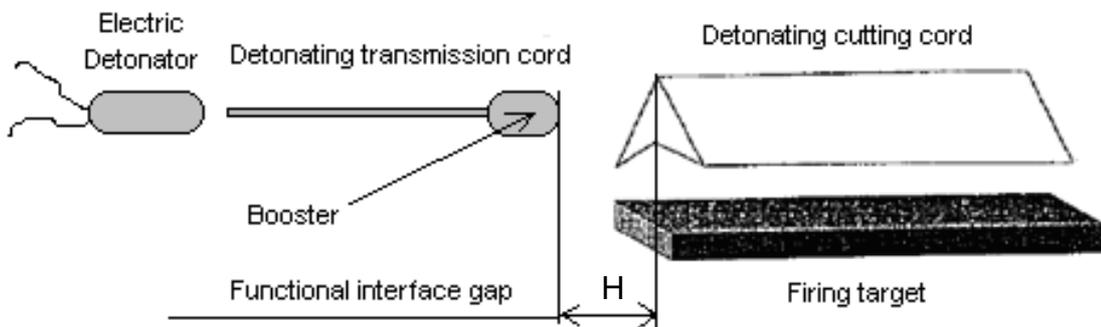
The system here is a detonating cutting cord triggered by a flexible detonating transmission cord.

NB: All the numerical data in this example are fictitious: they are provided as a case study to allow the reader to repeat the calculations.

D.1 OBJECTIVE

The objective is to validate the reliability of triggering detonation of the functional interface between the flexible detonating transmission cord and the cutting cord.

Transmission of the detonation wave from the flexible cable to the cutter is confirmed by the imprint left by the detonation created when the cord cuts a firing target.



D.2 IDENTIFYING THE SEVERITY PARAMETER

The physical parameter which triggers the cutting cord is the energy transmitted by the flexible detonating transmission cord.

Among the various functional parameters affecting this energy (explosive density in the line and the cord, functional gap, cord thickness, temperature, etc.), the severity parameter chosen is the triggering energy, which is related to the gap between the flexible detonating transmission cord and the cutting cord: a specific prior study and feedback for this type of interface have shown that there is a non-linear relationship between gap and initiation energy.

The gap H becomes the predominant parameter, because it allows varying the energy while satisfying the criteria of section C.2.3, namely:

- it has a significant effect on the performance to be achieved,
- it is quantifiable,
- it is measurable,
- it is easily adjustable by modifying the base height of the line in the interface equipment.

A parametric study is carried out on the gap to determine the level of energy imparted to the cutting cord as a function of the change in the gap between the

GTPS N°11F

flexible detonating transmission cord and the cutting cord: this situation establishes the relationship between the energy (the severity parameter) and the gap (experimental means for increasing severity of the energy parameter).

The severity parameter is calculated from the ratio of:

- the energy in the nominal configuration at the tolerance limits of the definition (X_{ref}),
- to the energy-severity parameter obtained by increasing the gap (X_{sev}).

D.3 DETERMINING THE COEFFICIENT OF VARIATION CVG

The functional parameter “detonation energy of the donor” depends on the following important parameters:

- the linear charge of the flexible detonating transmission cord (donor),
- the linear charge in the detonating cutting cord (receiver),
- the geometry of the end of the flexible detonating transmission cord,
- the geometry of the thickness of the cutting cord envelope,
- the gap **H** between booster and cutting cord.

The dispersion (**CVc**) is a consequence of the dispersion levels associated with the influential parameters:

- | | |
|--|-----------------|
| ➤ geometric dispersion of the gap H | CV0 = 3% |
| ➤ dispersion of the linear charge in the flexible detonating transmission cord | CV1 = 3% |
| ➤ dispersion of the linear charge in the cutting cord | CV2 = 3% |
| ➤ geometric dispersion of the end of the flexible detonating transmission cord | CV3 = 7% |
| ➤ geometric dispersion of the thickness of the cutting cord envelope | CV4 = 5% |

The elementary coefficients of variation **CVi** of these parameters are established from manufacturing and inspection tolerances for these parameters, assumed equal to ± 3 sigma.

Dispersion levels related to ageing, mechanical environments or thermal cycling are deliberately excluded from the calculation by applying the input specification.

Furthermore, the batches of equipment used for the trials are assumed to be representative of the population.

CVc is calculated as follow:

$$CVc = \sqrt{Cv0^2 + Cv1^2 + Cv2^2 + Cv3^2 + Cv4^2} = 10,05\%$$

Following recommendation 1 in section C.2.4.4 the calculated coefficient of variation CVc is increased by 10% to obtain the global coefficient of variation:

$$CVg = 1.1 \times CVc = 11.05\%$$

This value is acceptable conforming to recommendation 2 (section C.2.4.4):

$$3\% < CVg < 15\%$$

D.4 DESIGNING THE SEVERE TESTS PROGRAMME

The severity coefficient is "multiplier" (K_m) as the selected parameter (gap H) is inversely related to the detonation triggering energy.

The severe tests programme is built as follows:

- reliability target: **$R = 0.999$**
- confidence level: **$1-\alpha = 90\%$**
- coefficient of variation **$CV_g = 11.05\%$**
- number of severe trials: **$n = 5$**

The multiplier coefficient is therefore **$K_m = 1.46$** : this still satisfies recommendation 2 in section C.2.4.4 ($K_m \geq 1.2$).

D.5 IMPLEMENTATION AND EXPLOITATION OF SEVERE TESTS

Several cases may arise:

- **No failure** observed: nominal case
- **1 failure** observed but unexplained: this case is degraded but still usable
- **2 failures** observed but unexplained: this is a failure of the severe tests programme

NB: In cases where one or two failures are observed, it is assumed that these failures are not attributed to a cause related to manufacture of the samples concerned and/or to the test conditions (See table in section C2.6.2 and flowchart in **annexe 7**): these failures are attributed to the "residual statistical risk".

D.5.1 CASE 1: NO FAILURE OBSERVED

If the 5 severe trials are carried out without failure, then the reliability target for the functional interface between the flexible detonating transmission cord and the cutting cord is validated at **R=0,999** with a confidence level **1- α = 90%**.

D.5.2 CASE 2: ONE FAILURE OBSERVED (DEGRADED CASE)

If the 5 severe trials are carried out with **one** observed failure (the 5th trial under recommendation n°7 in section C.2.6.3), then the reliability target for the functional interface between the flexible detonating transmission cord and the cutting cord is not validated.

There are then two options:

- **Option 1** : it operates as such the test plan with its failure in degrading the reliability and/or the confidence level. In this case, the reliability can be evaluated:
 - **R= 0,9914** with a confidence level of **1- α = 90%**
 - **R= 0,999** with a confidence level of **1- α = 60%**
 - **R= 0,9964** with a confidence level of **1- α = 80%**
- **Option 2** : we complete the test plan with additional specimens to find the target level of reliability. In this case, it will add to the five tests already carried out:
 - **4** without failure additional tests to find the target level of reliability (**R=0,999** with **1- α = 90%**), either a total of 9 tests hardened (including 1 failure),

The **2nd option** is quite risky, because if the observed failure is not really due to the statistical risk as assumed, then the risk of a second failure in the new specimens tested remains strong.

D.5.3 CASE 3: TWO FAILURES OBSERVED (FAILURE OF THE SEVERE TESTS PROGRAMME)

If the 5 severe trials are carried out with two observed failures, then the reliability target for the functional interface between the flexible detonating transmission cord and the cutting cord is not validated.

This case is considered as a failure of the severe tests programme, and it is recommended that it should not be used 'as is'.

Indeed, if this programme with two failures were used, the following results would be obtained:

- **Option 1** : it operates as such the test plan with its two failures in degrading the reliability and/or the confidence level. In this case, the reliability can be evaluated:
 - **R= 0,9618** with a confidence level of **1- α = 90%**
 - **R= 0,999** with a confidence level of **1- α = 26%**
 - **R= 0,9935** with a confidence level of **1- α = 60%**
- **Option 2** : the tests plan is completed with additional specimens to find the reliability target. In this case, it will add to the five trials already carried out:
 - **8** additional trials without failure to find the reliability target (**R=0,999** with **1- α = 90%**), therefore a total of 13 severe trials (including 2 failures).

It can be observed that:

- the first option gives a resultant reliability very far from the target,
- the second option becomes prohibitive in terms of number of trials to be carried out; the risk of having other failures among the new samples tested becomes unacceptable, since it is very probable that the two observed failures are not due to the statistical risk only.

E. CONCLUSION

This recommendation describes the procedure to implement the severe tests method.

The method is applicable for assessment of a probability of success (or failure) during operation of a single-shot device. It outlines the procedure to be followed (actual conditions for implementation) emphasising in particular:

- selection of a predominant parameter, around which the severe trials will be carried out,
- determination of the coefficient of variation of the predominant severity parameter,
- determination of the severity coefficient and in consequence, the level at which the severe trials will be made,
- performance of the severe tests which must not, as a rule, produce any failure,
- exploitation of the results, especially when failures have been observed.

It enables an *a priori* target reliability level to be assessed from a few samples (< 10), assuming that the operational limits of the predominant functional parameter used are Normally distributed.

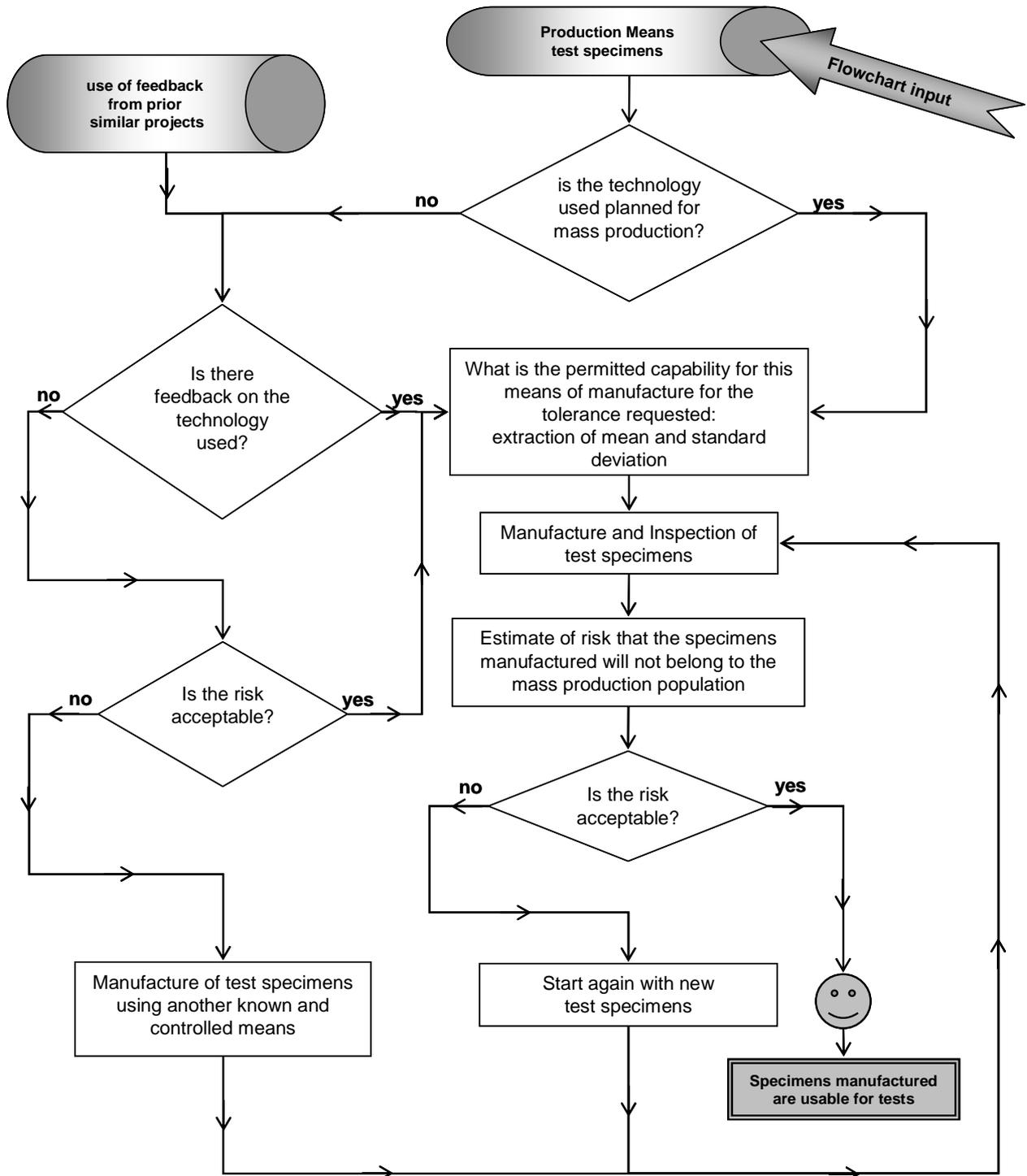
This evaluation of reliability, extended to an infinite population and corresponding to a given confidence level, puts a figure on the probability of the one-shot device operating correctly under the effect of a defined reference level of excitation.

F. ACKNOWLEDGEMENTS

The GTPS reliability commission would like to thank MM. HEYMANN and GOND (MBDA) in particular for their help in designing the method of severe testing with failures taken into account.

ANNEXES:

Annexe 1: Checking how representative the tested samples are



Annexe 2: CV weighting using experimental design

Calculation of the corrected coefficient of variation CV_c as given in section C.2.4.3 relies on the fact that all the elementary coefficients of variation CV_i are assigned an equal "weight": All the parameters associated with these CV_i play an equally important role in the performance being studied.

Use of the experimental design method avoids introducing such a bias into the calculations. The following methodology is proposed:

1/ Factor analysis

Identification of the predominant factors:

- Predominant factors (e.g.: thickness, distance, etc.): controllable and mutually independent; direct effect on the product; not comparable with so-called natural dispersion levels
- Environmental factors (temperature, etc.): scarcely or not controllable; all these factors generate a Normal performance distribution

2/ Design and implementation of an experimental protocol

The experimental design is to be implemented for all the predominant factors at the limits of usage of the product.

3/ Exploiting the experimental design by multilinear regression

Obtain a mathematical model of the product of type:

$$Y = A_0 + \sum_{j=0}^{k-1} A_{j+1} \times X_j$$

Where **A_j**: influence coefficient corresponding to the predominant factor X_{j-1} obtained from the experimental design.

X_j: predominant factor of the system

Y: performance being investigated

4/ Determination of total dispersion

The global coefficient of variation CV_g can then be obtained by concurrent calculation of the residual standard deviation obtained from the experimental design, and the coefficients of variation associated with the weighted predominant factors as a function of their "weights" according to the following mathematical model:

$$C_i = \frac{\partial Y}{\partial X_i} \Big|_{X_j = cte = \mu_j, \forall j \neq i}$$

Where **C_i**: effect coefficient

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The total dispersion is then obtained by a quadratic composition of the standard deviations with their effect coefficients C_i :

$$\sigma_{total} = \sqrt{\left[\sum_i C_i^2 \cdot \sigma(X_i)^2 \right] + \sigma_{residual}^2}$$

Where: $\sigma(X_j)$: standard deviation associated with the predominant factor X_j (obtained in accordance with section C.2.4.2)

$\sigma_{residual}$: residual standard deviation obtained from the experimental design (influence of "environmental" factors)

σ_{total} : total standard deviation related directly to CVg

The total standard deviation enables calculation of CVg: $CVg = \sigma_{total} / m_{performance}$

5/ Application example

Equipment: ram operating by shock wave

- Predominant factors:
 - X_0 (mass of secondary explosive) = 10 where $\sigma(X_0) = 0.1$
 - X_1 (mass of primary explosive) = 4 where $\sigma(X_1) = 0.2$
- Model: $Y = A_0 + A_1X_0 + A_2.X_0.X_1$ where $\sigma_{residual} = 0.05$
 - $A_0 = 2,5$
 - $A_1 = 0,5$
 - $A_2 = 0,2$

- Effect coefficients:

$$C_0 = \left. \frac{\partial Y}{\partial X_0} \right|_{X_1=\mu_1} = A_1 + A_2 \cdot \mu_1 = 0.5 + 0.2 \times 4 = 1.3$$

$$C_1 = \left. \frac{\partial Y}{\partial X_1} \right|_{X_0=\mu_0} = A_2 \cdot \mu_0 = 0.2 \times 10 = 2.0$$

Hence:

- Mean of the performance: **mean (Y)** = $2.5 + 0.5 \times 10 + 0.2 \times 10 \times 4 = 15.5$.
- Global coefficient of variation: **CVg** = $0.424/15 = 3 \%$.

Annexe 3: Treatment of test programmes with failures

Starting hypothesis: Implementation of a programme of **n** severe tests with **k** failures

The minimum probability of occurrence **P** in this programme of severe tests at a confidence level of **1-α** follows an incomplete Beta distribution:

$$I(P, n - k, k + 1) = \alpha$$

From this test programme, to prove that the probability of proper operation is at least equal to **R_s** (reliability at severe level **X_{sev}**), then the following relation must be satisfied:

$$P \geq R_s$$

This implies that: $I(R_s, n - k, k + 1) \leq \alpha$

The resolution of this inequality (see document ref. 20) allows us to calculate the reliability **R_s** at the severe level **X_{sev}**:

$$R_s = \left(I^{-1}(\alpha, n - k, k + 1) \right)$$

As noted in section C.2.5.3, the terms of the multiplier and divisor severity coefficients are:

$$K_m = \frac{1 + U_s \times CV_g}{1 + U_r \times CV_g} \quad \text{and} \quad K_d = \frac{1 - U_r \times CV_g}{1 - U_s \times CV_g}$$

For the multiplier coefficient, **R = N(-U_r)** and **R_s = N(-U_s)**; therefore the expression of severity coefficient **K_m** is:

$$K_m = \frac{1 - CV_g \cdot N^{-1} \left(I^{-1}(\alpha, n - k, k + 1) \right)}{1 - CV_g \cdot N^{-1}(R)}$$

For the divisor coefficient, **R = N(U_r)** and **R_s = N(U_s)**; therefore the expression of severity coefficient **K_d** is:

$$K_d = \frac{1 + CV_g \cdot N^{-1}(R)}{1 + CV_g \cdot N^{-1} \left(I^{-1}(\alpha, n - k, k + 1) \right)}$$

NOTE: It should be noted that these formulas (K_d and K_m) work for severe tests plans without failure (just enter 0 for the number of failures k).

These formulas are easily programmed in a spreadsheet (Microsoft EXCEL® for example): GTPS also provides an EXCEL spreadsheet running with the French version of Microsoft EXCEL®. Under EXCEL, $I^{-1}(\alpha, n - k, k + 1)$ is programmed as follows:

BETA.INVERSE (α ; n-k ; k+1 ; 0 ; 1)



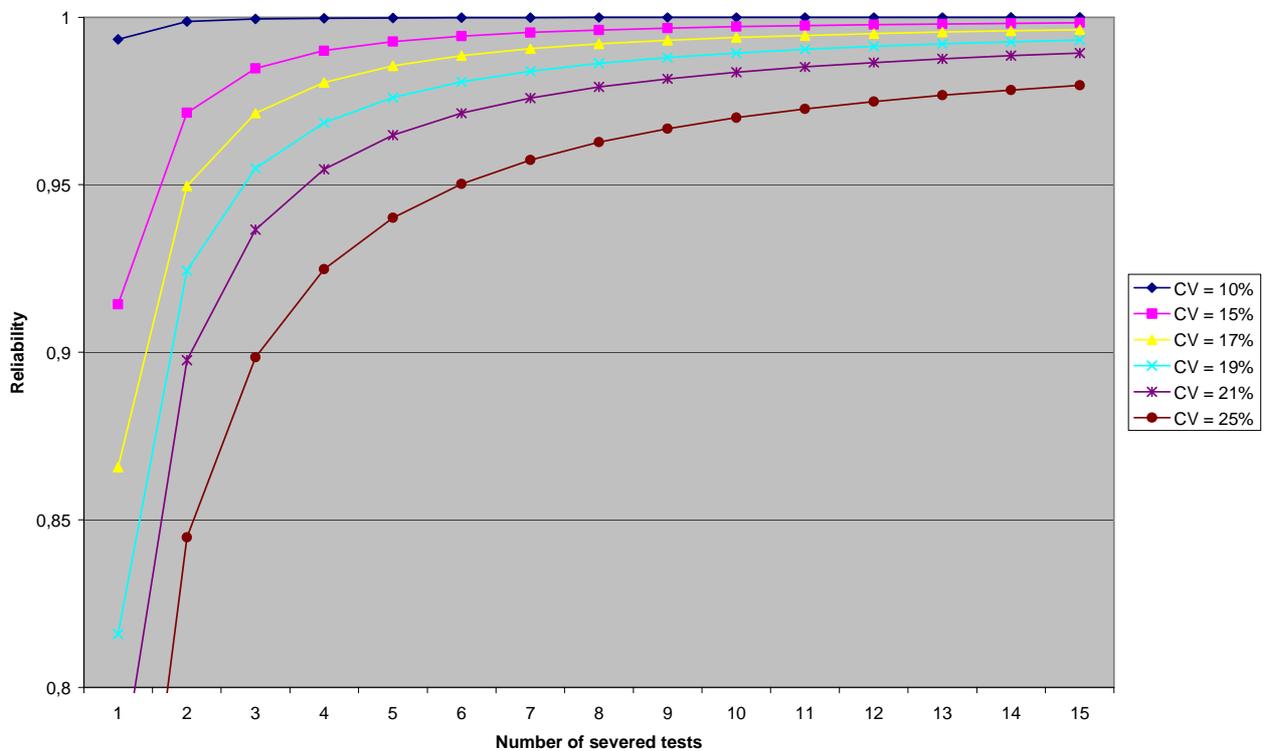
Annexe 4: Analysis and construction of CVg

Maximum recommended levels of CVg:

Use of a value of CVg > 15% is not recommended (see Recommendation 2, section C.2.4.4). The graphs below illustrate this recommended limit concept.

Graph 1 shows that high reliability levels are hard to reach with values of CVg greater than 15% (as in the case of a multiplier severity coefficient Km of 1.5):

- for 4 successful tests, the reliability level obtained remains below 0.990
- for 15 successful tests, the reliability level obtained remains below 0.998

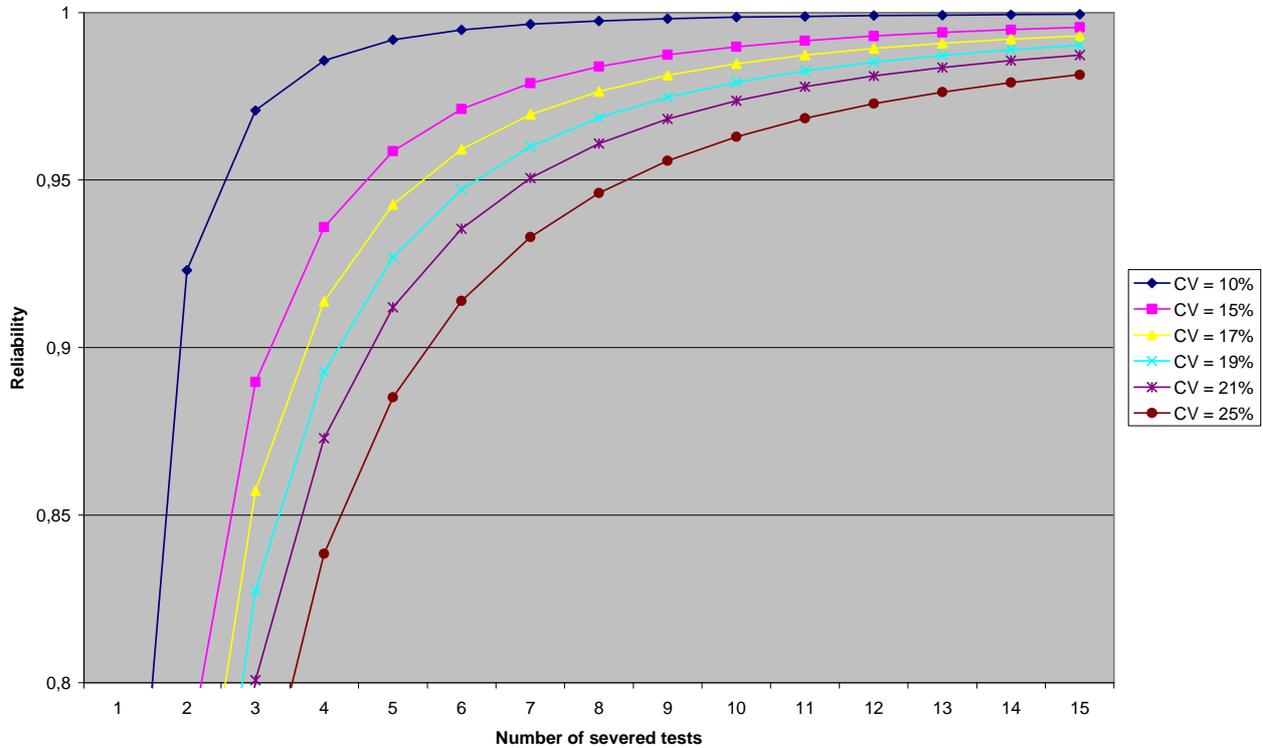


Graph 1 – reliability levels with a confidence level of 90% as a function of the number of tests with a multiplier severity coefficient Km of 1.5

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Graph 2 shows that high reliability levels are hard to attain with values of CVg greater than 15% (as in the case of a divisor severity coefficient Kd of 1.2):

- for 4 successful tests, the reliability level obtained remains below 0.950
- for 15 successful tests, the reliability level obtained remains below 0.995

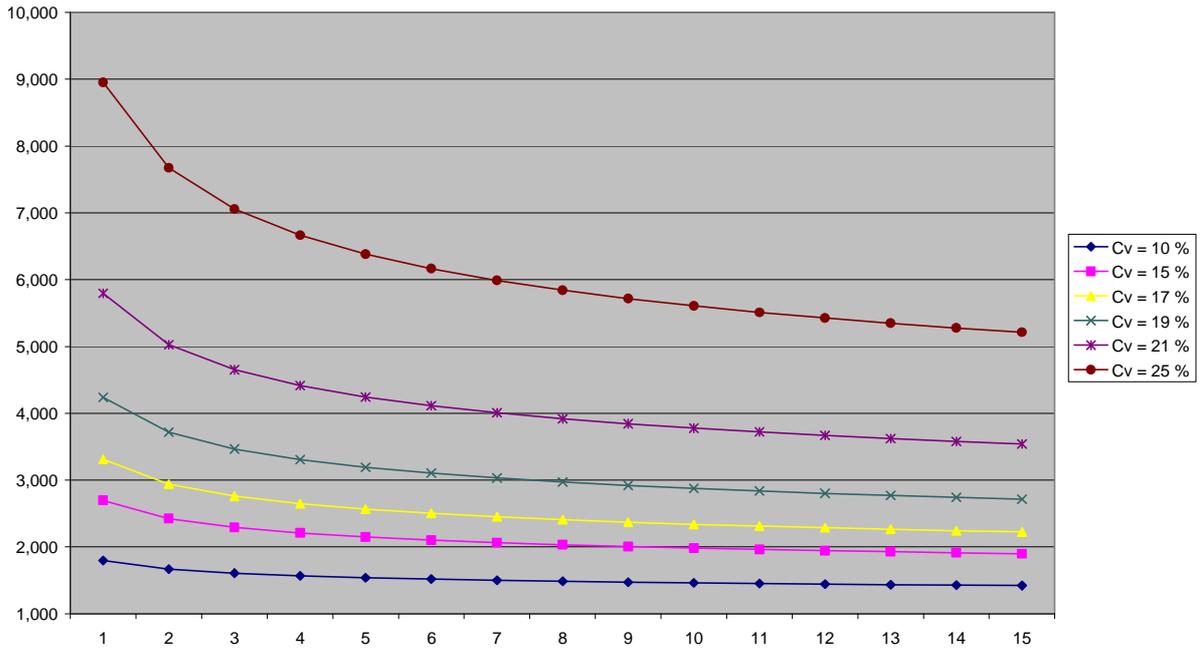


Graph 2 – reliability levels with a confidence level of 90% as a function of the number of tests with a divisor severity coefficient Kd of 1.2

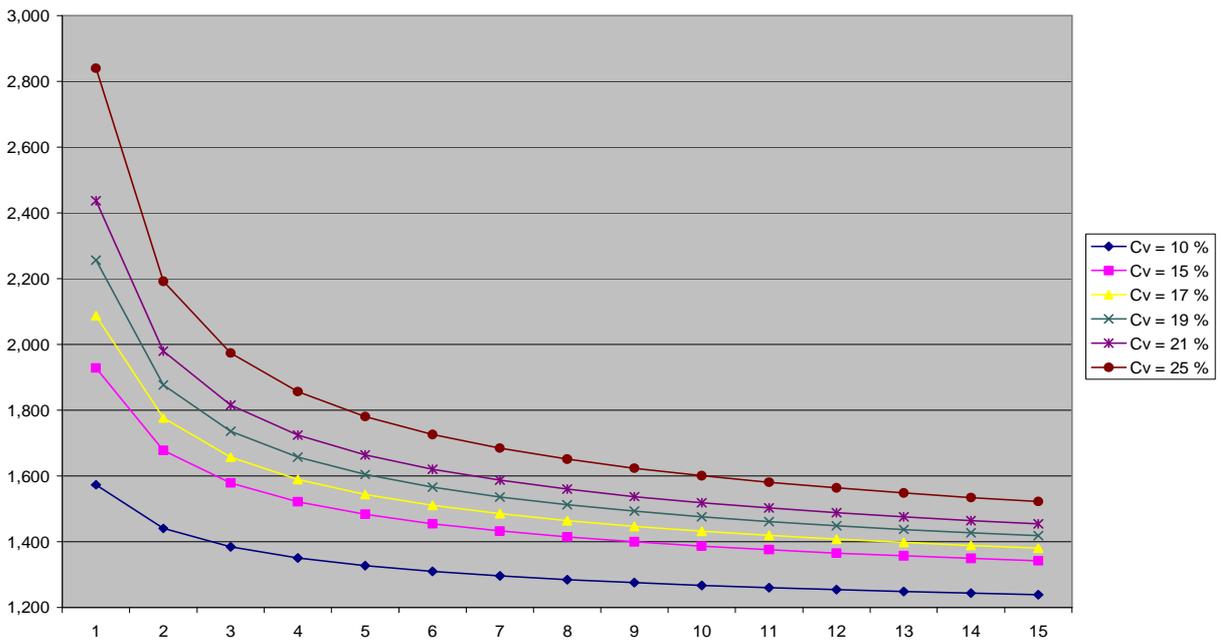
The same observation applies when setting the reliability objective and calculating the requisite severity coefficient as a function of the number of tests (see graphs 3 and 4, next page).

The requisite levels of severity coefficients for the same reliability objective then become quite high, potentially inconsistent with the definition or implementation.

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Graph 3 – Multiplier severity coefficient as a function of number of tests, with a reliability objective of 0.9999 for a confidence level of 90%



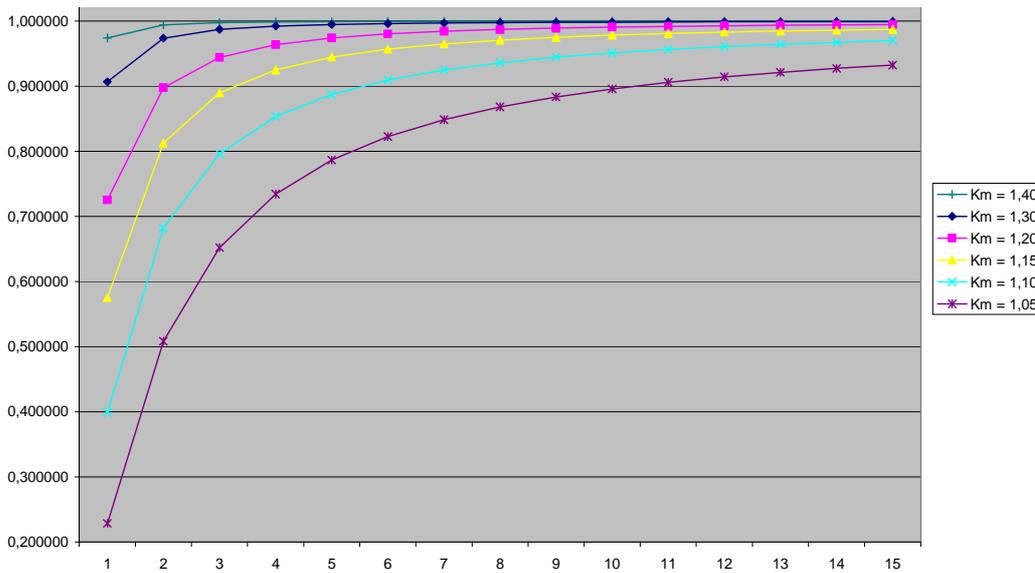
Graph 4 – Divisor severity coefficient as a function of number of tests, with a reliability objective of 0.9999 for a confidence level of 90%

Annexe 5: Minimum level of severity coefficient

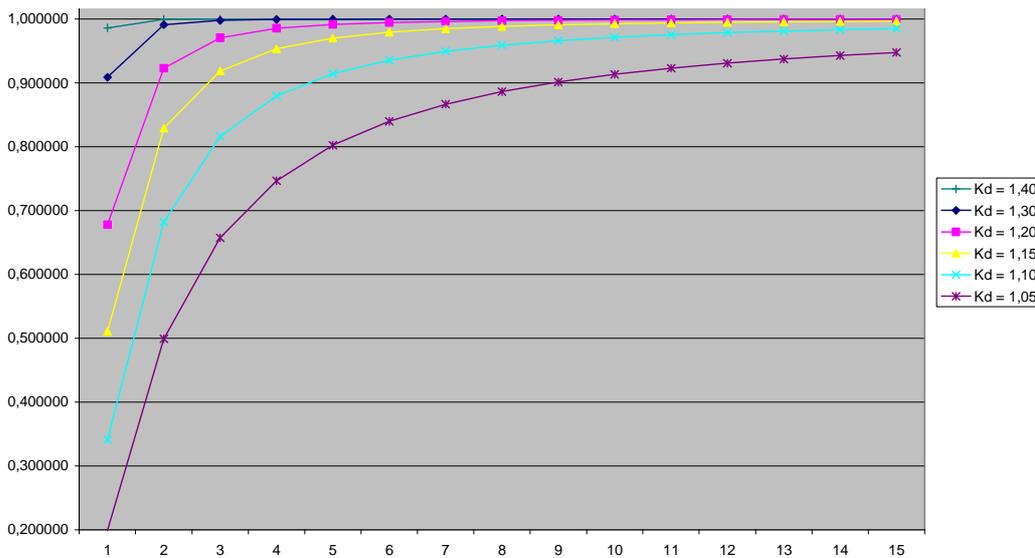
To guarantee a sufficient margin between the reference level and the severe level (see Recommendation 5, section C.2.5.5), a severity coefficient greater than 1.2 is recommended. The graphs below illustrate this recommended limit concept.

Graphs 5 and 6 show that high reliability levels are hard to attain with severity coefficients less than 1.2:

- for 5 successful tests, the reliability level obtained remains below 0.995 with $K_m = 1.15$
- for 11 successful tests, the reliability level obtained remains below 0.995 with $K_d = 1.15$

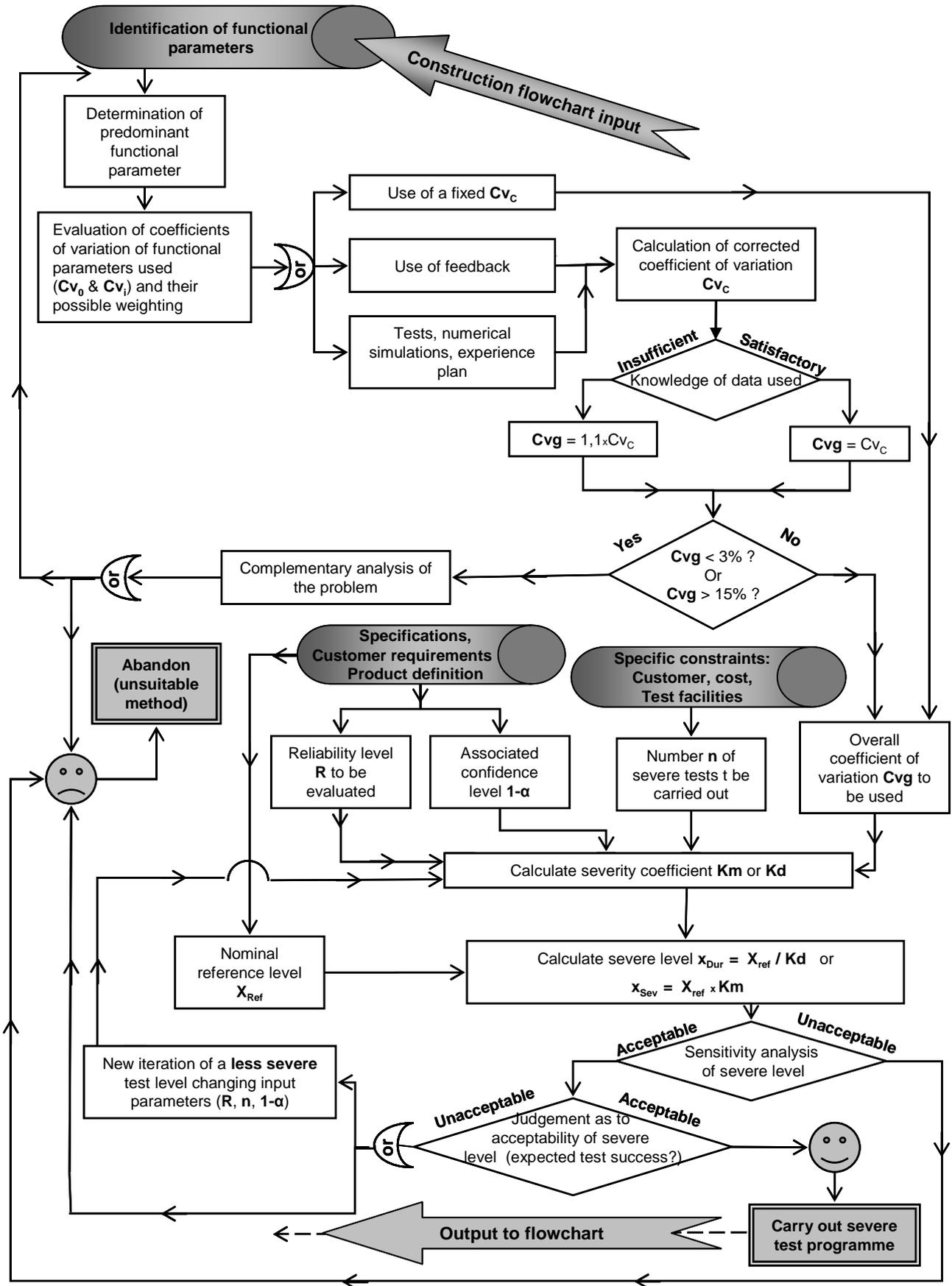


Graph 5 – Reliability levels ($1-\alpha = 90\%$ & $CVg = 10\%$) as a function of the number of tests (multiplier coefficient K_m)

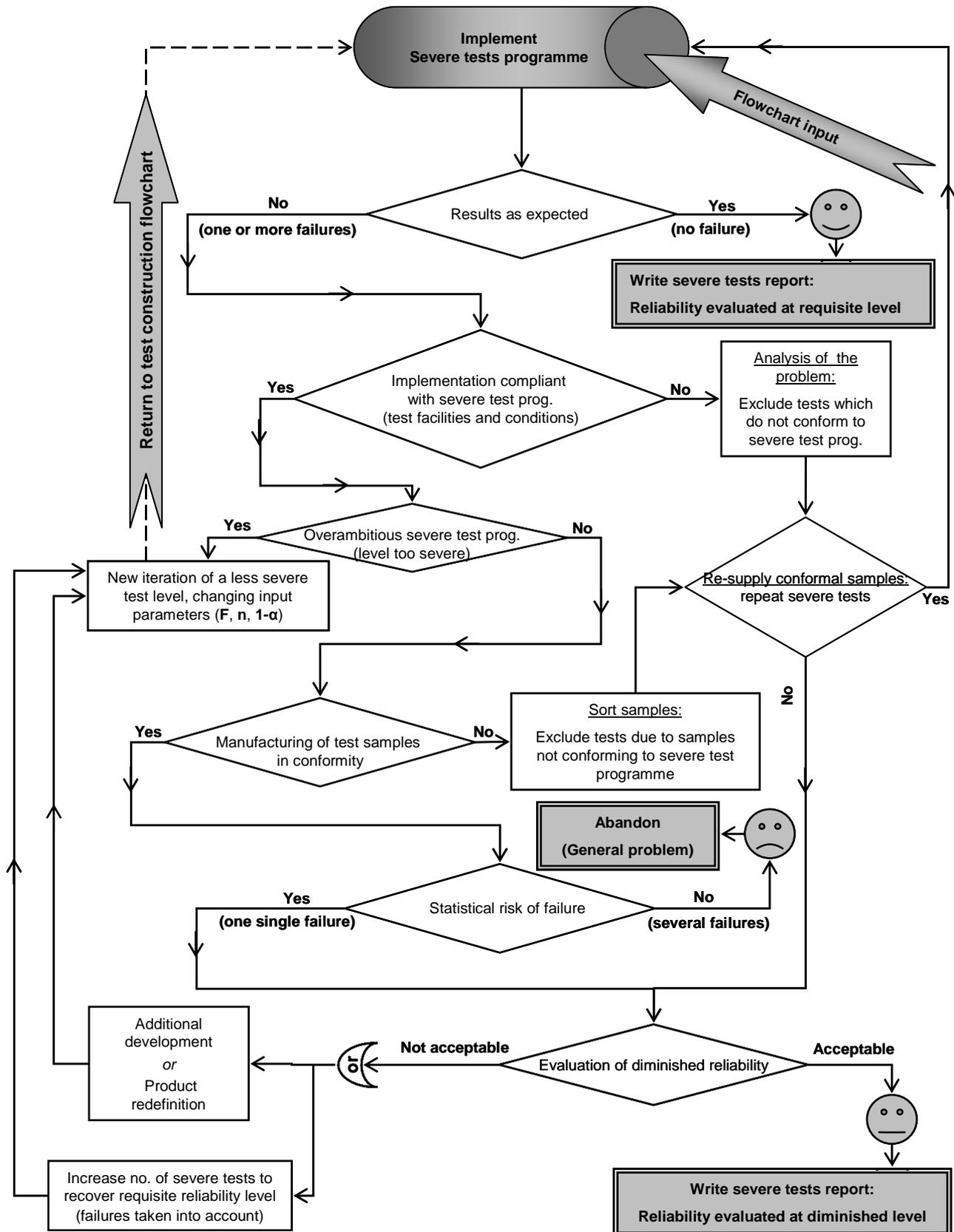


Graph 6 – Reliability levels ($1-\alpha = 90\%$ & $CVg = 10\%$) as a function of the number of tests (divisor coefficient K_d)

Annexe 6: Construction of severe tests programme



Annexe 7: Implementation and use of the severe tests programme



Annexe 8: Sensitivity analysis of the severity coefficient

All the numerical data in this example are fictitious: they are provided as a case study to allow the reader to repeat the calculations.

We wish to realise a severe tests programme with the current required to trigger an electrical igniter as the predominant parameter.

The target reliability level is:

- $R = 0.99999 = 1-10^{-5}$
- $1-\alpha = 90\%$

For economical reasons, only two severe firings can be performed.

The level of precision in firing current adjustment is 0.1 A: this is much lower than the standard deviation of the functional parameter.

The functional characteristics of this parameter are as follows:

- Minimum current required for ignition: $X_{ref} = 5A$
- Coefficient of variation: $CVg = 15\%$ (i.e. standard deviation $\sigma = 0.75A$)

The calculated divisor severity coefficient is then: $Kd = 1.766$

This Kd gives a severity level of: $X_{sev} = 5/1.766 = 2.83A$.

If the reliability level is increased to $R = 0.999999 = 1-10^{-6}$, the severe tests programme would then become:

- $Kd = 1.854$
- $X_{sev} = 2.71 A$

The difference between these two calculated severe levels is only 0.12 A: this is of the same order of magnitude as the adjustment precision for the firing current; in other words, using the same test allows to evaluate very different levels of reliability.

If the reliability level is $R=0.9999 = 1-10^{-4}$, then the test programme would be:

- $Kd = 1.68$
- $X_{sev} = 2.98 A$

i.e. a difference of 0.15 A between the two levels: We notice that, with a Cv of 15% and for two severe tests, the limits of the method are reached for a reliability of $R=0.9999$ at a confidence level of $1-\alpha=90\%$.

